Ritt's Algorithm and Differential Resultants of Two Algebraic ODE's

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Abstract

Let R be a differential ring with a derivation $\{\delta\}$ and let $R\{x\}$ be the ring of differential polynomials in x with coefficients in R.

Let f_1, f_2 be differential polynomials in $R\{x\}$ respectively of order m and n.

In 1932 Ritt introduced the notion of differential resultant of two such differential polynomials with some additional hypothesys as the irreducible and primitive differential polynomial in the coefficients of f_1 and f_2 and their derivatives, that is obtained by his algorithm for an extended characteristic set of the differential ideal $[f_1, f_2]$. Such definition disappeared in the successive papers and books of Ritt and there is nothing about the order and the degree of such polynomial.

In 1995 the author introduced the notion of differential resultant of f_1 and f_2 as the algebraic resultant in the definition of Macaulay of the algebraic polynomials f_1 and f_2 and their derivatives until order respectively n and m. This definition allows to have an upper bound on the order and the degree of each coefficient of f_1 and f_2 appearing in such differential polynomial. Furthermore no additional hypothesys is riquired on f_1 and f_2.

A relation between such two notions is studied in this paper. In particular it is proved that both these resultants are in the ideal generated by $\delta^n f_1, \ldots, \delta f_1, f_1, \delta^m f_2, \ldots, \delta f_2, f_2$ and the bound m+n on the order is effective.

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